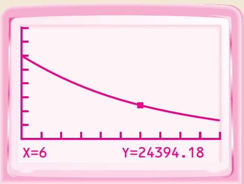
Extra Example 4

Annual sales of a certain product can be modeled by the function $S = 60{,}000e^{-0.15t}$, where S is the number of units sold and t is the number of years since the product went on the market.

• Graph the model.



• Use the graph to estimate the annual sales 6 years after the product went on the market. **about 24,400 units**

Avoiding Common Errors

When students see the equation $y = ae^{rx}$ that defines the natural base exponential function in the Key Concept box on page 493, they may think that e is a variable because they are used to letters representing variables and because there are several other variables in the equation. Make sure that they understand that e is in fact a constant—a specific irrational number like π.

Mathematical Reasoning

Students may be puzzled by the concept of continuously compounded interest. Ask students if the result of continuously compounding of interest will be much different from daily compounding. Use this as an opportunity to discuss informally the idea of a limiting process: As the frequency of compounding increases, the process gets closer and closer to continuous compounding. Tell students that the study of limiting processes is important in more advanced mathematics, especially calculus.

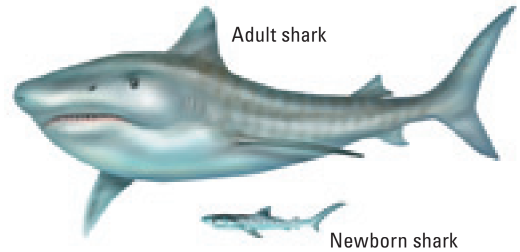
EXAMPLE 4 Solve a multi-step problem

BIOLOGY The length ℓ (in centimeters) of a tiger shark can be modeled by the function

$$\ell = 337 - 276e^{-0.178t}$$

where t is the shark's age (in years).

- Graph the model.
- Use the graph to estimate the length of a tiger shark that is 3 years old.



INTERPRET VARIABLES

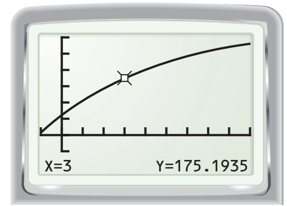
On a graphing calculator, enter the function $\ell = 337 - 276e^{-0.178t}$ using the variables x and y, as shown below: $y = 337 - 276e^{-0.178x}$

Solution

STEP 1 **Graph** the model, as shown.

STEP 2 **Use** the *trace* feature to determine that $\ell \approx 175$ when $t = 3$.

▶ The length of a 3-year-old tiger shark is about 175 centimeters.



✓ GUIDED PRACTICE for Examples 3 and 4

Graph the function. State the domain and range. **6–8. See margin.**

6. $y = 2e^{0.5x}$

7. $f(x) = \frac{1}{2}e^{-x} + 1$

8. $y = 1.5e^{0.25(x-1)} - 2$

9. **WHAT IF?** In Example 4, use the given function to estimate the length of a tiger shark that is 5 years old. **about 224 cm**

CONTINUOUSLY COMPOUNDED INTEREST In Lesson 7.1, you learned that the balance of an account earning compound interest is given by this formula:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

As the frequency n of compounding approaches positive infinity, the compound interest formula approximates the following formula.

KEY CONCEPT *For Your Notebook*

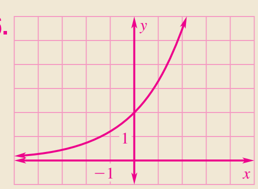
Continuously Compounded Interest

When interest is compounded *continuously*, the amount A in an account after t years is given by the formula

$$A = Pe^{rt}$$

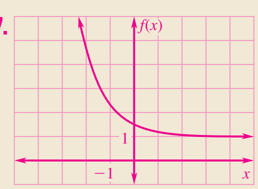
where P is the principal and r is the annual interest rate expressed as a decimal.

6.



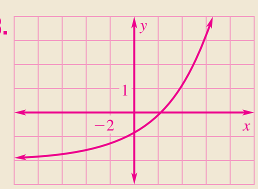
domain: all real numbers, range: $y > 0$

7.



domain: all real numbers, range: $y > 1$

8.



domain: all real numbers, range: $y > -2$

EXAMPLE 5 Model continuously compounded interest

FINANCE You deposit \$4000 in an account that pays 6% annual interest compounded continuously. What is the balance after 1 year?

Solution

Use the formula for continuously compounded interest.

$A = Pe^{rt}$ **Write formula.**

$= 4000e^{0.06(1)}$ **Substitute 4000 for *P*, 0.06 for *r*, and 1 for *t*.**

≈ 4247.35 **Use a calculator.**

▶ The balance at the end of 1 year is \$4247.35.

GUIDED PRACTICE for Example 5

10. **FINANCE** You deposit \$2500 in an account that pays 5% annual interest compounded continuously. Find the balance after each amount of time.

    a. 2 years **\$2762.93** **b.** 5 years **\$3210.06** **c.** 7.5 years **\$3637.48**

11. **FINANCE** Find the amount of interest earned in parts (a)–(c) of Exercise 10.

7.3 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 35, and 57

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 16, 52, 53, and 60

SKILL PRACTICE

A

1. **VOCABULARY** Copy and complete: The number _?_ is an irrational number approximately equal to 2.71828. ***e***

2. ★ **WRITING** Tell whether the function $f(x) = \frac{1}{3}e^{4x}$ is an example of *exponential growth* or *exponential decay*. Explain. **Exponential growth; the power is $4x$ and is greater than 0 therefore the function is an exponential growth function.**

EXAMPLE 1 on p. 492 for Exs. 3–18

SIMPLIFYING EXPRESSIONS Simplify the expression.

3. $e^3 \cdot e^4$ e^7
4. $e^{-2} \cdot e^6$ e^4
5. $(2e^{3x})^3$ $8e^{9x}$
6. $(2e^{-2})^{-4}$ $\frac{e^8}{16}$
7. $(3e^{5x})^{-1}$ $\frac{1}{3e^{5x}}$
8. $e^x \cdot e^{-3x} \cdot e^4$ e^{-2x+4}
9. $\sqrt{9e^6}$ $3e^3$
10. $e^x \cdot 5e^{x+3}$ $5e^{2x+3}$
11. $\frac{3e}{e^x}$ $3e^{1-x}$
12. $\frac{4e^x}{e^{4x}}$ $\frac{4}{e^{3x}}$
13. $\sqrt[3]{8e^{9x}}$ $2e^{3x}$
14. $\frac{6e^{4x}}{8e}$ $\frac{3e^{4x-1}}{4}$

15. ★ **MULTIPLE CHOICE** What is the simplified form of $(4e^{2x})^3$? **C**

    (A) $4e^{6x}$ (B) $4e^{8x}$ (C) $64e^{6x}$ (D) $64e^{8x}$

16. ★ **MULTIPLE CHOICE** What is the simplified form of $\sqrt{\frac{4(27e^{13}x)}{3e^7x^{-3}}}$? **D**

    (A) $6e^{10}x$ (B) $6e^6x^4$ (C) $\frac{6e^3}{x^2}$ (D) $6e^3x^2$

Extra Example 5

You deposit \$3000 in an account that pays 3.5% annual interest compounded continuously. What is the balance after 3 years? **\$3332.13**

Key Question to Ask for Example 5

- What formula could you use to calculate just the amount of interest earned on an account in which interest is compounded continuously? $I = Pe^{rt} - P$

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: When is the natural base e useful?

- **As n approaches $+\infty$, $\left(1 + \frac{1}{n}\right)^n$ approaches the irrational number e, which is approximately 2.718.**
- **The formula for the amount A in an account when interest is compounded continuously is $A = Pe^{rt}$, where P is the principal, r is the annual interest rate expressed as a decimal, and t is the time in years.**

The natural base e is a special irrational number. This base is used in many applications of exponential functions, including continuously increasing or decreasing biological phenomena and continuously compounded interest.